VINAYAK CLASSES
DEGREE & DIPLOMA ENGINEERING

Sub: Maths

Standard: **12th (Science)** **Date:**
Time : **3 Hours** **Total Marks: 80**

Note:

(1) All questions are compulsory.
(2) The question paper consist of 30 questions divided into four sections A, B, C, D
(3) Section-A contains 6 Multiple Choice Questions of 1 mark each.
Section –B contains 8 questions of 2 marks each. (One of them will have internal option)
Section-C contains 6 questions of 3 marks each. (Two of them will have internal option)
Section-D contains 10 questions of 4 marks each. (Three of them will have internal option)
(4) Use of logarithmic tables is allowed.
(5) Use of calculator is not allowed.
(6) In LPP only rough sketch of graph is expected. Graph paper is not compulsory.

SECTION-A (6 Marks)

Select and write the correct answer from the given alternatives in each of the following questions:

Q.1 If the points $A(2,1,1), B(0,-1,4) and\ C(k,3,-2)$ are collinear, then $k = \cdots$

(a) 0 (b) 1 (c) 4 (d) -4

Q.2.Which of the following is logically equivalent to: $\sim[\sim p \rightarrow q]$

(a) $p v \sim q$ (b) $\sim p \Lambda q$ (c) $\sim p \Lambda q$ (d) $\sim p\ \Lambda \sim q$

Q.3 The lines $\frac{x}{-1} = \frac{y}{2} = \frac{z}{2}\ and\ \frac{x}{2} = \frac{y}{2} = \frac{z}{-1}\ are$ ...

(a)parallel (b) skew lines (c) perpendicular (d) None of these

Q.4 If $f(x)$ is continuous at $x = 0$, where

$f(x) = \left(\frac{5x+1}{3x+1}\right)^{\frac{1}{x}}$, when $x \neq 0$

$= k$, when $x = 0$

Then the value of k is

(a) 0 (b) e^2 (c) e^3 (d) $\frac{1}{e^3}$

Q.5 Assume that a spherical raindrop evaporates at the rate proportional to surface area. Differential equation involving rate of change of radius of raindrop is…

(a) $\frac{dv}{dt} = -k$ (b) $\frac{dr}{dt} = -rk$ (c) $\frac{dr}{dt} = -k$ (d) $\frac{d}{dt}(2\pi r^2) = k$

Q.6 In a binomial distribution with $n = 4$ $and\ if$ $2\,P(X = 3) = 3P\,(X = 2)$ then value of p is ….

(a) $\frac{9}{13}$ (b) $\frac{4}{13}$ (c) $\frac{6}{13}$ (d) $\frac{7}{13}$

SECTION-B (16 Marks)

Q.7.Find the general solution of $\cos\left(x + \frac{\pi}{10}\right) = 0$

Q.8. In ΔABC, prove that $a(b\cos C - c\cos B) = b^2 - c^2$

Q.9. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

OR

Q.9. Find the Cartesian equation of the line passing through the points $A(3,4,-7)and\ B(6,-1,1)$

Q.10. Write the following statement in symbolic form and find its truth value.

$\forall n \in .N. n^2 + n$ is an even number and $n^2 - n$ is an odd number.

Q.11. Find $\frac{dy}{dx}$ $if x\sin y + y\sin x = 0$

Q.12. The displacement s of a particle at a time t is given by $s = t^3 - 4t^2 - 5$. Find its velocity and acceleration at $t = 2$.

Q.13. Find the area of region laying in first quadrant and bounded by $y = 4x^2, y = 2$ and $y = 4$.

Q.14. The probability that a bomb will hit a target is 0.8. Find the probability that out of 10 bombs dropped, exactly 4 will hit the target.

SECTION-C (18 Marks)

Q.15. Using truth table examine whether the statement pattern $(p \wedge q)v(p \wedge r)$ is a tautology, contradiction or contingency.

Q.16. If from a point Q (a, b, c) perpendicular QA and AB are drawn to YX' and ZX planes respectively, then find the vector equation of the plane OAB.

Q.17. If angle between vectors $\bar{a}$ *and* $\bar{b}$ having direction ratio's 1, 2, 1 and 1.3k, 1 is $\frac{\pi}{4}$ then find k.

OR

Q.17. If M is foot of perpendicular from $P(2, 4, 3)$ on the line joining the points A(1, 2, 4) and B(3, 4, 5), find co-ordinate of M.

Q.18. The p.m.f. of r.v.X is $P(x) = \frac{1}{15}$, for $x = 1,2, \ldots 14,15$

= 0, otherwise. Find (i) E(X) (ii) Var (X)

Q.19. If u and v are two functions of x, then prove that $\int uv\, dx = u\int v\, dx - \int \left[\frac{du}{dx}\int v\, dx\right] dx$

OR

Q.19. Evaluate $\int \frac{1}{3-2\cos 2x} dx$

Q.20. Discuss the continuity of the following functions, at $x = 0$.

$f(x) = \frac{x}{|x|}$, for $x \neq 0$

$=1$, for $x = 0$

SECTION-D (40 Marks)

Q.21. Solve the following equations by method of reduction.

$x - y + z = 4, 2x + y - 3z = 0, x + y + z = 2$

Q.22. Show that: $cos^{-1}\left(\frac{4}{5}\right) + cos^{-1}\left(\frac{12}{13}\right) = cos^{-1}\left(\frac{33}{65}\right)$

Q.23.Minimize $z = 4x + 5y, subject\ to\ 2x + y \geq 7, 2x + 3y \leq 15, x \leq 3, x \geq 0, y \geq 0.$ Solve using graphical method.

Q.24. Find $\frac{dy}{dx}\ if\ y = x^2 + (\sin x)^x + \log x$

OR

Q.24. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}\ where\ \frac{dy}{dx} \neq 0. Hence\ find\ \frac{d}{dx}(tan^{-1} x).$

Q.25. Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represent a pair of lines, Also find acute angle between them.

Q.26. Evaluate: $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$

Q.27. Evaluate $\int \frac{d\theta}{\sin\theta+\sin 2\theta}$

Q.28.Using vector method, find the incentre of the triangle whose vertices are P(0, 4, 0), Q(0, 0, 3) and R(0, 4, 3).

OR

Q.28. Prove that the volume of the parallelepiped with coterminous edges as $\bar{a},\bar{b},\bar{c}\ is\ [\bar{a}\bar{b}\bar{c}]$ and hence find the volume of the parallelepiped with its coterminous edges $2\bar{\imath}+5\bar{\jmath}-4\bar{k}, 5\bar{\imath}+7\bar{\jmath}+5\bar{k}\ and\ 4\bar{\imath}+5\bar{\jmath}-2\bar{k}$

Q.29 Solve the differential equation $(1-x^2)\frac{dy}{dx}+2xy=x(1-x^2)^{1/2}$

OR

Q.29. Reduce the differential equation to $(x-y)^2\frac{dy}{dx}=a^2$ to variable separable form and hence solve.

Q.30. Find the approximate value of $log_{10}(1016)$ given that $log_{10}e=0.4343$.